HEAT TRANSFER AT LARGE PRANDTL NUMBERS IN THE WALL REGION WITH PRESSURIZATION

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A steady turbulent flow is parallel to an infinite permeable plane wall through which is injected a liquid having the same physical properties as the liquid of the main flow. The generalized hypothesis of localizability of turbulent transport is used for analysis. The Stanton number is calculated in a wide range of variation of the Prandtl number for different values of the Reynolds number and a parameter characterizing pressurization.

Liquids, representing organosilicon compounds, are widely used at present in various areas of the national economy. They are also used [1] as damping liquids in special coatings intended for quenching turbulent fluctuations for the purpose of reducing friction during movement of bodies in water. The Prandtl numbers of such liquids reach an order of a thousand owing to low thermal diffusivity, whereas their viscosity coefficients change within very wide limits [2] and permit completely the existence of a turbulent flow regime at a turbulent flow regime at moderately high speeds.

Therefore, a calculation of heat transfer in a turbulent flow at large Prandtl numbers and the decrease of heat transfer at the wall are of practical value.

We will consider the turbulent flow of an incompressible liquid parallel to an infinite permeable plane wall. A liquid having the same physical properties as the liquid of the main flow is injected through the wall.

It is assumed that the injection velocity of the liquid is small in order to provide the stable existence of a turbulent flow. For the values of the Prandtl number being considered the velocity of pressurization has an order of magnitude not greater than the order of the transverse velocities of the flow. This permits us to use in the analysis the hypothesis of localizability of turbulent transport, generalized by L. G. Loitsyanskii [3] for this case, when the molecular and molar processes in the wall region are quantitatively comparable to one another. The generalized hypothesis of localizability permits obtaining the main characteristics of the flow in a continuous form as integrals which we can easily calculate numerically by assigning a certain value of the parameter characterizing pressurization.

The flow is assumed steady in a dynamic and thermal sense. The equations of momentum and energy for the flow being considered have the form

$$\rho v \frac{\partial u}{\partial y} = \frac{\partial}{\partial y} (\rho \varepsilon + \mu) \frac{\partial u}{\partial y} = \frac{\partial \tau}{\partial y}, \tag{1}$$

$$\rho c_p v \frac{\partial T}{\partial y} = \frac{\partial}{\partial y} (\rho \varepsilon + \mu) c_p \frac{\partial T}{\partial y} = \frac{\partial q}{\partial y}. \tag{2}$$

The equation of continuity gives

$$\partial v / \partial y = 0, \tag{3}$$

i.e., the transverse velocity component is constant across the flow, $v = \text{const} = v_W$, where v_W is the injection velocity of the liquid through the wall. Consequently, Eqs. (1) and (2) can be written so:

$$\frac{\partial}{\partial y} (\tau - \rho v_w u) = 0, \tag{4}$$

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$$\frac{\partial}{\partial y}(q-\rho c_p v_w T)=0 \tag{5}$$

or

$$\tau-\rho v_w u=\text{const}=\tau_w, \tag{6}$$

$$q-\rho c_p v_w T=\text{const}=q_w. \tag{7}$$

The use of the quantitative expression of the hypothesis of localizability enables us to write Eqs. (6) and (7) in the form

$$\tau=\tau_w+\rho v_w u=\mu\frac{du}{dy}f(R), \tag{8}$$

$$q=q_w+\rho c_p v_w T=\lambda\frac{d\theta}{dy}\left[1-\Pr+\Pr f(R)\right]. \tag{9}$$

In this case we mean by R the expression

$$R=\frac{Ul}{\nu}=\frac{l^2}{\nu}\cdot\frac{du}{dy},\; l=\varkappa y \tag{10}$$

or the expression more common from the standpoint of the localizability hypothesis

$$R=\frac{\varkappa^2}{\nu}\,\frac{\left(\frac{du}{dy}\right)^3}{\left(\frac{d^2u}{dy^2}\right)^2}. \tag{11}$$

The quantity f(R) represents the characteristic function of the interaction of molecular and molar exchange. In the entire flow region f(R) was used in the form [3]

$$f=1+R\left[1-\exp(-\alpha R)\right],\quad \alpha=0.0125\ [4]. \tag{12}$$

In universal coordinates Eqs. (8)-(10) are transformed to

$$1+\beta\varphi=\frac{d\varphi}{d\eta}f(R), \tag{13}$$

$$1+\beta\psi=\frac{d\psi}{d\eta}\left[\frac{1}{\Pr}-1+f(R)\right], \tag{14}$$

$$\eta=\frac{1}{\varkappa}\sqrt{Rf(R)}, \tag{15}$$

where

$$\varphi=u/v_*;\quad \psi=\theta/\theta_*;\quad \theta_*=q_w/\rho c_p v_*,\qquad \eta=yv_*/\nu,\quad v_*=\sqrt{\tau_w/\rho}, \tag{16}$$

$\beta = v_w/v_*$ is the parameter characterizing pressurization.

The solution of Eqs. (13)-(15) in a parametric form with R as a parameter has the form

$$\varphi=\frac{1}{\beta}\left\{\exp\left[\frac{\beta}{2\varkappa}\int_0^R\frac{Rf'(R)+f(R)}{f(R)\sqrt{Rf(R)}}\,dR\right]-1\right\}, \tag{17}$$

$$\psi=\frac{1}{\beta}\left\{\exp\left[\frac{\beta}{2\varkappa}\int_0^R\frac{Rf'(R)+f(R)}{\left(\frac{1}{\Pr}-1+f(R)\right)\sqrt{Rf(R)}}\,dR\right]-1\right\}. \tag{18}$$

Thus the universal distribution of velocities φ and the universal distribution of the temperatures ψ can be calculated numerically by assigning certain values of the parameter β.

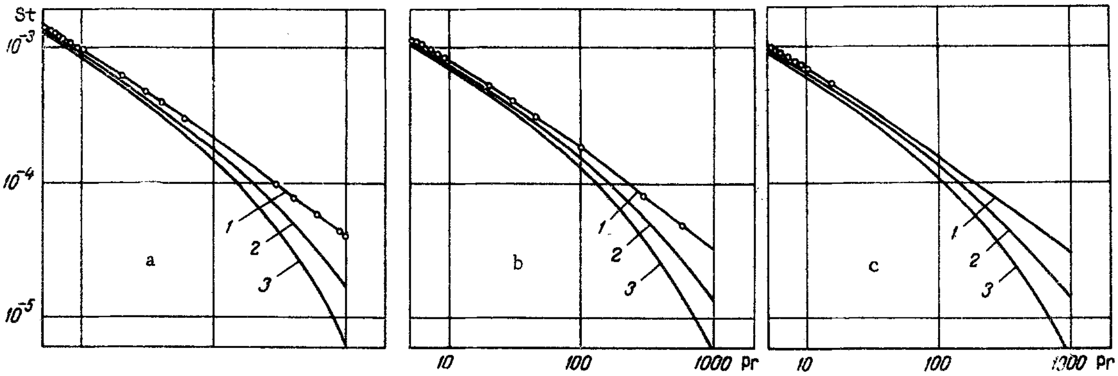

Fig. 1. Stanton number vs Prandtl number and pressurization parameter for Reynolds number: a) 10^4; b) $2.5 \cdot 10^4$; c) $5 \cdot 10^4$; c) $5 \cdot 10^4$. 1) $\beta = 0$; 2) 0.01; 3) 0.002.

The Stanton number is usually used as the total characteristic of the heat-transfer process

$$\mathrm{St} = \frac{\mathrm{Nu}}{\mathrm{Re}\,\mathrm{Pr}} = \frac{q_w}{\rho u_{av} c_p \theta_{av}}, \tag{19}$$

where u_{av} and θ_{av} are the average values of the velocity and temperature. According to (16) and (19)

$$\mathrm{St} = 1/\varphi_{av}\psi_{av}, \tag{20}$$

where

$$\varphi_{av} = \frac{1}{R_1 f(R_1)} \int_0^{R_1} \varphi\left(\sqrt{R_1 f(R_1)} - \sqrt{Rf(R)}\right) \frac{f + Rf'}{\sqrt{Rf(R)}}\, dR, \tag{21}$$

$$\psi_{av} = \frac{\int_0^{R_1} \psi\varphi\left(\sqrt{R_1 f(R_1)} - \sqrt{Rf(R)}\right) \frac{f + Rf'}{\sqrt{Rf(R)}}\, dR}{\int_0^{R_1} \varphi\left(\sqrt{R_1 f(R_1)} - \sqrt{Rf(R)}\right) \frac{f + Rf'}{\sqrt{Rf(R)}}\, dR}. \tag{22}$$

The Reynolds number, formed with respect to the average velocity u_{av} and pipe diameter d, is determined by the relation

$$\mathrm{Re} = 2\eta_1\varphi_{av} = 5\varphi_{av}\sqrt{R_1 f(R_1)} \tag{23}$$

(η_1 is the quantity η on the axis of the pipe).

Using Eqs. (20)-(22), we express the Stanton number by the integral

$$\mathrm{St} = \frac{R_1 f(R_1)}{\int_0^{R_1} \psi\varphi\left[\sqrt{R_1 f(R_1)} - \sqrt{Rf(R)}\right] \frac{f + Rf'}{\sqrt{Rf(R)}}\, dR}, \tag{24}$$

where φ and ψ are determined respectively by Eqs. (17) and (18).

Thus we calculated the Stanton number for different values of the Reynolds number and parameter β, the Prandtl number having varied in a wide range from 5 to 1000. The curves of the Stanton number as a function of the Prandtl number for a given Reynolds number and $\beta = 0.001$ and 0.002 that were obtained are presented in Fig. 1a, b, c. Analogous curves obtained in [4] in the absence of injection of a liquid through the wall, i.e., for $\beta = 0$, are also presented there. The circles mark the experimental data obtained by Deissler [5] on heat transfer in a turbulent flow of liquids with large Prandtl numbers in the absence of pressurization, i.e., for $\beta = 0$.

We see from the figure that, by means of a very small injection of liquid through the wall, it is possible to reduce heat transfer considerably, especially in liquids whose Prandtl numbers are sufficiently large.

As far as we know there are no experimental data on heat transfer during turbulent flow of liquids with large Prandtl numbers in the presence of pressurization.

NOTATION

u, v	are the longitudinal and transverse components of the average velocity of the flow;
y	is the transverse coordinate;
T	is the temperature of the flow;
θ	is the difference between the temperature of the surface and flow;
ρ	is the density of liquid;
μ, ν	are the dynamic and kinematic viscosities of liquid;
ε	is the turbulent mixing coefficient;
τ, τ_w	are the shear stress and shear stress at wall;
q	is the heat flux;
q_w	is the heat flux through wall;
c_p	is the heat capacity;
v_w	is the injection velocity of liquid through wall;
l	is the mixing length $\varkappa = 0.4$;
λ	is the thermal conductivity of liquid;

$Pr = \mu c_p/\lambda$;
$Re = u_{av} d/\nu$.

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